Nonlinear Finite Element Analysis of Fibrous Reinforced Concrete Beam-Column Joints

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Abstract

In this paper, the finite element method is used to study the nonlinear behaviour of beam-column fibrous reinforced concrete joints under short–term monotonic loading. Concrete is represented by eight nodded isoparametric elements and the reinforcement was represented by axial two noded bar elements embedded in the concrete elements. Strain hardening approach, has been employed to model the compressive behavior of the fibrous concrete. In tension a continuous function is used to model fibrous concrete in the pre-peak and post – peak states. Material nonlinearities due to cracking of concrete, crushing of concrete in compression, debonding and pull – out of fibers and yielding of reinforcement have been taken into account. A smeared fixed crack approach of the cracked concrete in tension is assumed. An incremental – iterative scheme based on Newton – Raphson's method is employed for the nonlinear solution algorithm and a displacement criterion is adopted for checking the convergence of the solution. Several previously published test results for fiber reinforced concrete beam-column joints were analyzed and the numerical results showed good agreement with the published experimental results.

Keywords: Beam, Column, Fibrous concrete, Finite element, Joint, Nonlinear analysis.

التحليل غير الخطي بالعناصر المحددة لعقد عتبة-عمود خراسانية ليفية مسلحة

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الخلاصة

استعملت طريقة العناصر المحددة في هذا البحث لدراسة السلوك غير الخطي لعقد عتبة- عمود خرسانية ليفية مسلحة تحت تأثير أحمال قصيرة الأمد متزايدة. مثلت الخرسانة بعناصر رباعية موحدة المعملية ذات ثمانية عقد والتسليح بعناصر أحادية المحور ذات عقدتين مطمورة داخل عناصر

الخرسانة. استعمل نموذج تصلب الانفعالات لتمثيل سلوك الخرسانة الليفية في الانضغاط. كما استعملت دالة متصلة في الشد لتمثيل الخرسانة الليفية قبل وبعد التشقق. كما أخذت سلوك المواد غير الخطية بنظر الاعتبار مثل تشقق الخرسانة، سحق الخرسانة في الانضغاط، انسحاب الألياف من الخرسانة وخضوع التسليح. افترض نموذج الشق الثابت المنتشر. استعملت طريقة الزيادة- التكرار وحسب طريقة نيوتن-رافسون للحل غير الخطي كما استعمل معيار الإزاحة لتدقيق تقارب الحل. تم تحليل عدة عقد عتبة-عمود من الخرسانة الليفية المسلحة وقورنت النتائج مع نتائج عملية منشورة وأظهرت تقاربا جيدا.

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Introduction

Formerly, the design of monolithic reinforced concrete joints was limited to providing adequate anchorage for the reinforcement. However the increasing use of high strength concrete and steel with the present strength design method results in a reduction of cross-sectional dimensions and this require more attention to be paid to the joint design.

The basic requirement at joints is that all of the existing forces at the ends of the members must be transmitted through the joint to the supporting members. In such cases the joint must be designed to resist forces that the beams and columns transfer to that joint and these may include axial loads, bending moments, torsion and shear forces. Joint loads and forces resulting from gravity or gravity and lateral loads (wind or earthquake forces) will develop a pattern of diagonal cracking due to diagonal tensile stresses that result from the interaction of axial , bending and shear forces.

The interior joints are normally confined by the beams in the two orthogonal directions while an exterior or corner joint require confinement reinforcement in a form of column ties [1] placed within the joint between the top and bottom beams reinforcement. The provision of such ties especially in relatively small beam and column sections is rather difficult. Steel fibers may play the role of this confinement reinforcement, enhancing the tensile strength of concrete and thus retarding diagonal cracks initiation and increasing the nominal shear strength of the joint [2].

In this paper a nonlinear finite element analysis for monolithic fibrous reinforced concrete beam column joints is presented so that it can be used instead of conducting experimental tests which should include so many parameters; such as reinforcement area and their detail, concrete and steel strengths and steel fibres properties. Only Planar joints are considered, since experimental test results for spatial joints are not available in the literature.

Finite Element Formulations

Since only planar joints are considered in this study, the beam-column joints were assumed in a state of plane stress. Shear is the most predominant effect in joints and for this reason concrete is represented by eight nodded isoparametric elements; since such elements can simulate shear deformation better than four nodded elements. Formulation of this element and the corresponding matrices can be found in reference [3]. The main reinforcing bars, ties and stirrups are represented by an axial two nodded elements embedded anywhere within the concrete element. A perfect bond is assumed between concrete and the reinforcement; this means that the concrete and steel have the same strain value at any common point. Formulation of these bar elements can be found in reference [4]

The combined incremental – iterative technique is used in the present study for solving the equilibrium equation. In this method the tangential stiffness matrix has to be updated at the second iteration within each load increment and maintains the same matrix for the successive iterations until convergence is achieved. A displacement convergence criterion is used in this study as employed by many investigators. The convergence is considered to occur when the norms of the incremental displacements are within a given tolerance of the norm of the total displacements as follows:

$$\sqrt{\sum_{k=1}^{N_d} \Delta d_k^i \Delta d_k^i} \Big/ \sqrt{\sum_{k=1}^{N_d} d_k^i d_k^i} \leq TOL \qquad \text{……………………………………..(1)}$$

where N_d is the total degrees of freedom, Δd_k^i and d_k^i are the incremental and total i^{th} displacement respectively and TOL is a prescribed tolerance. A tolerance of 3% was assumed in the present investigation.

Materials Constitutive Relationships For Fibrous Concrete

Compressive Stress-Strain Relationship

In the present study a strain–hardening approach was adopted, with a linear portion that starts from zero to 30 percent of the compressive strength followed by a parabolic stress–strain curve until the effective stress reach the compressive strength of fibrous concrete, then a perfect plastic behaviour is assumed until crushing occurs as shown, Fig.(1). The strain ε_{pf} at peak stress f'_{cf} as given by Soroushian and Lee [5] is used in this investigation:

$$\varepsilon_{pf} = \frac{2f'_c}{E_c} + 0.0007\frac{v_f l_f}{d_f} \quad \text{...(2)}$$

where f'_c = cylinder compressive strength of plain concrete, v_f = fiber volume fracion and l_f and d_f = length and diameter of the fibers respectively. In the absence of test results, the following equation which was proposed in reference [5] can be used to estimate the compressive strength of fibrous concrete:

$$f'_{cf} = f'_c + 3.6 v_f . l_f / d_f \quad \text{..(3)}$$

The maximum compressive strain derived by Abdul-Razzak [4] is used in this investigation which is based on the experimental test results of Shah and Rangan [6]:

$$\varepsilon_{cuf} = 3011 + 2295 v_f \text{ (micro strain)..(4)}$$

Tensile Stress–Strain Relationship

A continuous function for the ascending and descending parts of the stress-strain curve in tension of plain concrete proposed by Carreira and Chu [7] is used in references [8, 9] for fibrous concrete, Fig.(1), in the following form and is adopted in this study:

$$\frac{f_t}{f'_{tf}} = \frac{\beta(\varepsilon/\varepsilon_{tf})}{\beta - 1 + (\varepsilon/\varepsilon_{tf})^{\beta}} \quad \textbf{...(5)}$$

where ε = the tensile strain at a stress f_t, f'_{tf} = tensile strength of fibrous concrete, ε_{tf} = strain at peak stress and β is a fibers parameter defined in references [10, 11] as follow:

$$\beta = 1.093 + 0.7132 R.I.^{-0.926} \quad \text{for hooked fibers ....(6a)}$$

$$\beta = 1.093 + 7.4848 R.I^{-1.387} \quad \text{for smooth fibers} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6b)$$

$$\beta = 0.5811 + 1.93 R.I^{-0.7406} \quad \text{for crimped fibers} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6c)$$

RI is a reinforcing index = $w_f l_f / d_f$ and w_f is the ratio of the weight of the fibers to that of concrete. In the absence of experimental data the strain ε_{tf} at peak stress f'_{tf} for fibrous concrete can be defined as proposed in reference [5] as follow:

$$\varepsilon_{tf} = \varepsilon_t \left(1 + 0.35 N_f \, d_f \, l_f\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$f'_{tf} = f'_t \left(1 + 0.016 N_f^{1/3} + 0.05\pi \, d_f l_f N_f\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where ε_t =cracking strain of plain concrete $= f'_t / E_c$, f'_t = tensile strength of plain concrete, E_c = elastic modulus of plain concrete and N_f = Number of fibers per unit area and is equal to $\eta_o \left(4V_f / \pi d_f^2\right)$, η_o is an orientation factor, for three dimensional distribution may be taken as 0.41 as derived in reference [12] or 0.5 as proposed by Hannant [13].

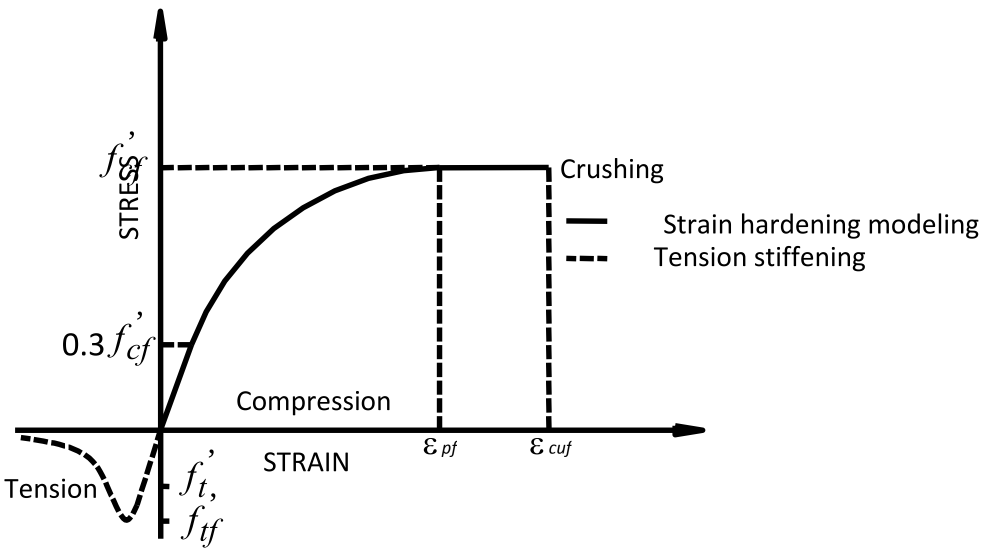


Fig.(1) Stress-strain relationships for fibrous concrete in uniaxial compression and tension

Biaxial Behaviour of Fibrous Concrete

In compression-compression state, plastic behaviour of concrete assumed to commence when the effective stress exceeds $0.3f'_{cf}$ and perfect plasticity assumed to occur when the effective stress exceeds f'_{cf}. A yield criterion was assumed to trace the plastic behaviour:

$$f(I_1, J_2) = \left[\beta_f (3J_2) + \alpha_f I_1\right]^{0.5} = f'_{cf} \qquad \text{...(9)}$$

where I_1 and J_2 are the first and second stress invariants and α_f and β_f are materials parameters derived in reference [4] from a wide range of steel fiber concrete mixes:

$$\alpha_f = \frac{1-\omega^2}{\omega^2 - 2\omega} f'_{cf} \ , \qquad \beta_f = \frac{1-2\omega}{\omega^2 - 2\omega} \qquad \text{...............................(10)}$$

and $\omega = e^x$ where:

$$x = \frac{1}{3.339 - 0.9772 \ln(v_f l_f / d_f)} \qquad \text{...................................(11)}$$

An isotropic hardening rule is assumed to define the uniform expansion of the yield surface.

In the tension-compression state, the following parabolic relationship that has been derived by Mahmood [14] is used in this investigation:

$$\sigma_2 / f'_{cf} + S^2 \left[\sigma_1 / f'_{cf}\right]^2 = 1 \quad \text{...(12)}$$

where $S = f'_{cf} / f'_{tf}$.

In the tension-tension state, no mutual interaction is assumed, i.e.; the tensile strength of concrete in the two orthogonal directions is assumed equal to that in the uniaxial state.

Shear Modulus of Cracked Concrete

A stress criterion is used for cracks initiation and for concrete cracked in direction 1, the shear modulus of the cracked concrete $\overline{G}$ proposed by Al-Mahaidi [15] for plain concrete and used later on in references [4,8] for fibrous concrete is used:

$$\overline{G} = 0.4G / (\varepsilon_1 / \varepsilon_{tf}) \quad \text{..(13)}$$

For concrete cracked in direction 2, $\overline{G}$ becomes:

$$\overline{G} = o.4G / (\varepsilon_2 / \varepsilon_{tf}) \quad \text{...(14)}$$

In this zone the degrading effect of the cracks on the compressive strength of concrete in the normal direction was taken into account according to the relationship proposed by Vecchio and Collins [16]:

$$\frac{f_{cf\,\max}}{f'_{cf}} = \frac{1}{0.8 + 0.34\left(\frac{\varepsilon_1}{\varepsilon_o}\right)} \leq 1.0 \qquad \text{.......................................(15)}$$

Stress-Strain Relationship of the Reinforcement

A bilinear relationship was assumed from zero stress to the yield strength with a slope E_s =200GPa followed by a straight line with a prescribed hardening modulus E_s'.

Numerical applications

Seven steel fibrous reinforced concrete beam- column joints have been analyzed by using the computer program. The numerical examples have been chosen in order to demonstrate the applicability of the programme in the numerical analysis of steel fibrous reinforced concrete beam-column joints.

Cross Type Beam –Column Connection

The first example presents the analysis of two reinforced concrete beam-column joints. The geometry, finite element mesh and details of the reinforcement are shown in Fig.(2). Taking advantage of symmetry, only one half of the beam-column joint was modeled. These specimens referred to as (C-1) and (CC-1) were tested by Sood and Gupta [17]. The materials properties specified in Ref.[17] and used in the analysis are summarized in Table (1). The failure loads are also shown in the Table.

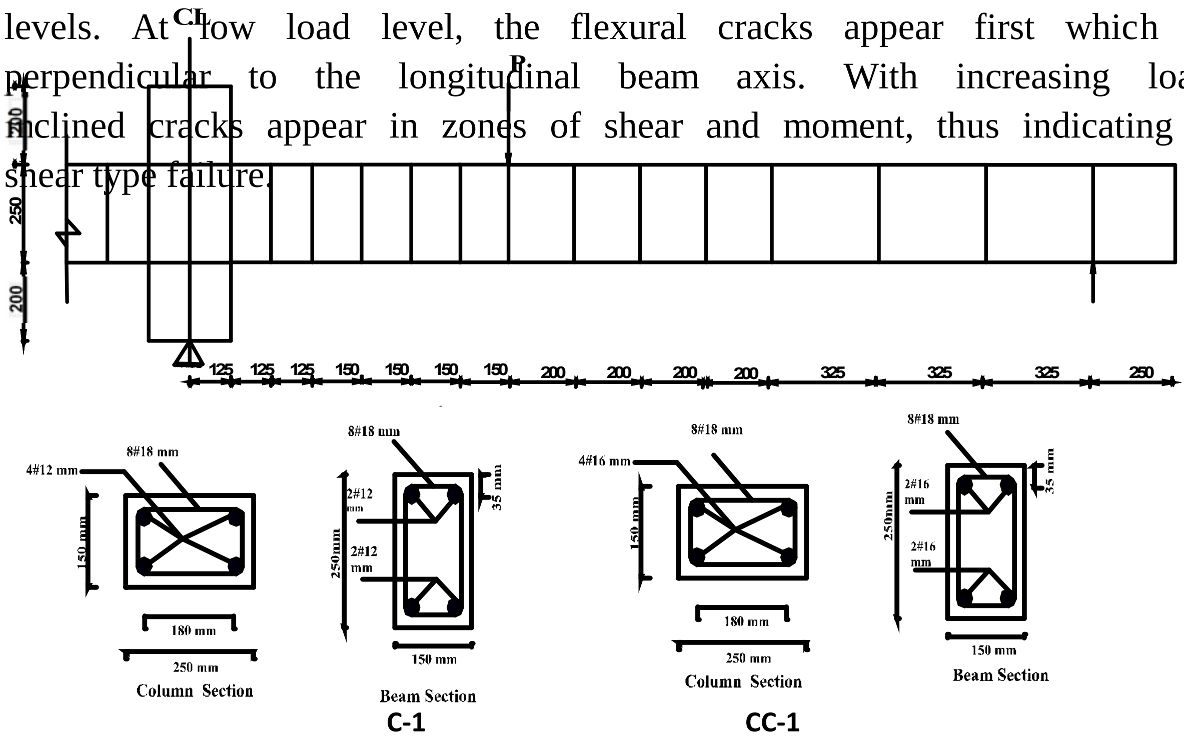
Table(1) Materials Properties of the Cross-type beam-column Connections Ref.[17]

Specimen No.	C-1	CC-1
Reinforcement	4#12	4#16

f_y (MPa)	500	510
Ties and Stirrups	18#8	18#8
f_y (MPa)	384	384
v_f% *	0.6	1.0
Cube Strength (MPa)	23.5	24.7
Experimental Failure Load (kN)	49.5	75.02
Numerical Failure Load (kN)	51.9	73.5

* $l_f = 31.5 mm$, $d_f = 0.315 mm$, (black annealed round mild steel fibres).

In Fig. (3) the load-deflection curves from the present analysis and the load deflection curve from Ref.[18] under load was represented. In Ref.[18] four nodded finite elements were used to model concrete and discrete bar elements for the reinforcement. The element used in this investigation is superior to that used in Ref.[18] in simulating shear failure. The predicted cracks pattern is shown in Fig. (4) for three load levels. At low load level, the flexural cracks appear first which is perpendicular to the longitudinal beam axis. With increasing load inclined cracks appear in zones of shear and moment, thus indicating a shear type failure.



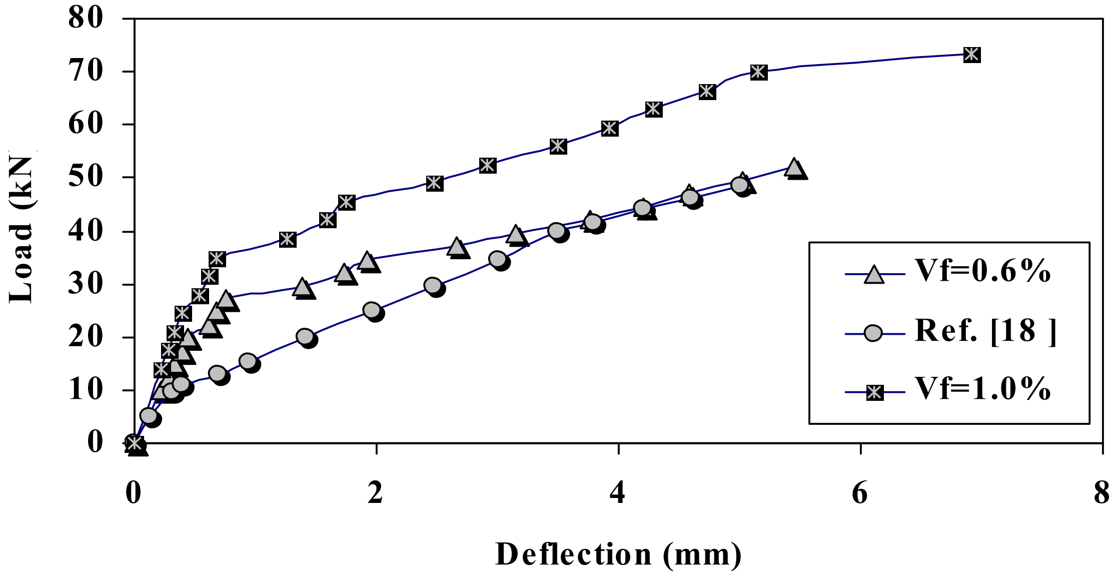


Fig.(3) Predicted Load – Deflection Curves for Cross Type Beam- Column Connection.

Fig.(3) Numerical Load-Dlection Curves for Cross type Beam-Column Connection.

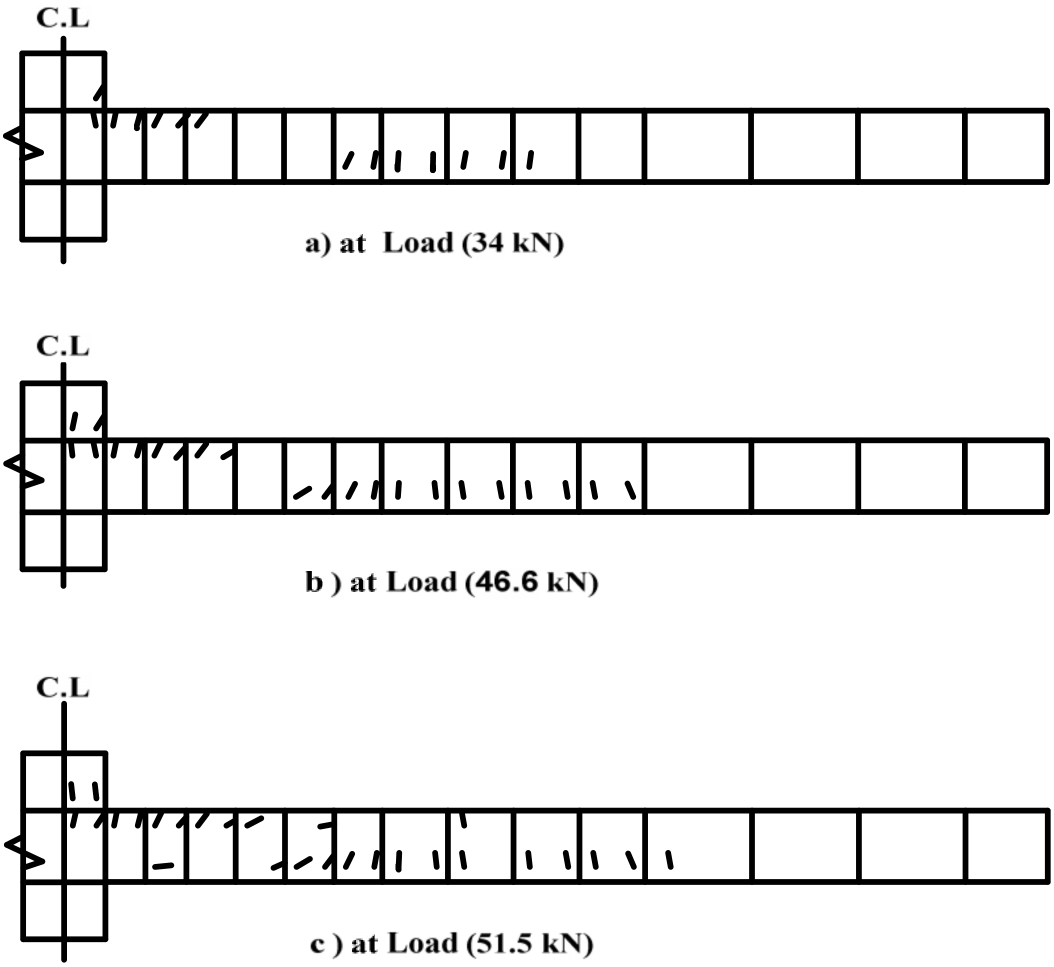
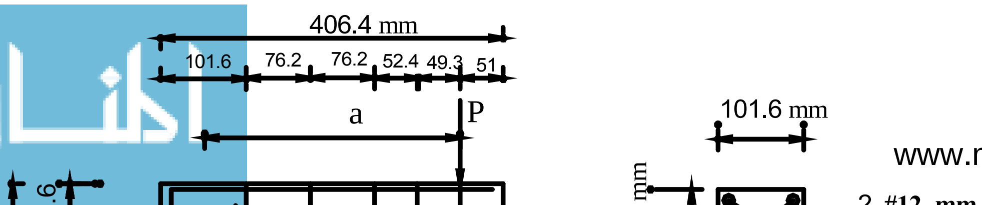


Fig.(4) Crack Pattern of Cross-Type Beam Column Connection at Loads (34, 46.6, and 51.5 kN).

L-Type Beam column Connection

Jindal and Sharma [19] tested 92 L-type steel fiber reinforced concrete Beam-Column connections to determine the effect of steel fiber on strength and behaviour of reinforced concrete joints. Three identical L-type connections were selected for the analysis. The 28-days compressive strength was 20.7MPa, Brass–Coated plain high strength circular steel fibers of size (25.4 ×0.254mm) and deformed steel bars having yield strength of (465.5 MPa) were used in both the beam and column sections. Plain steel bars having yield strength of (224) MPa were used as ties for the column section. The geometry, finite elements mesh



and load details with cross sections are shown in Fig. (5). The variable in these connections was the arm of the applied load which varies from 254 mm to 304.4 mm. In this study each connection is modeled by ten elements as shown in Fig. (5). The load versus vertical deflection curve under the load position is shown in Fig. (6). The comparison is restricted to the numerical and experimental failure loads, since the complete response of the joints is not reported in Ref.[19].

Table (2) shows the numerical and the experimental failure loads for three lever arms. A safe estimate of the failure load is obtained from the numerical results. Fig.(7) shows the crack pattern from the numerical analysis of the connection where the load is applied at an arm of 254 mm. Fig. (8) Shows the experimentally failed specimen and in most of the tested connections there are no cracks in the connection region, and failure occurred near the support by formation of cracks and large rotation. As quoted by Jindal and Sharma [19] in most of the cases, the test had to be discontinued due to opening of the cracks and large rotation near the support, resulting in large deflection under the load at the cantilevered end of the beam as shown in Fig. (8). It seems also that the steel connection at the specimens base was not rigid enough to act as a fixed support and this have influenced the point of fracture [19].

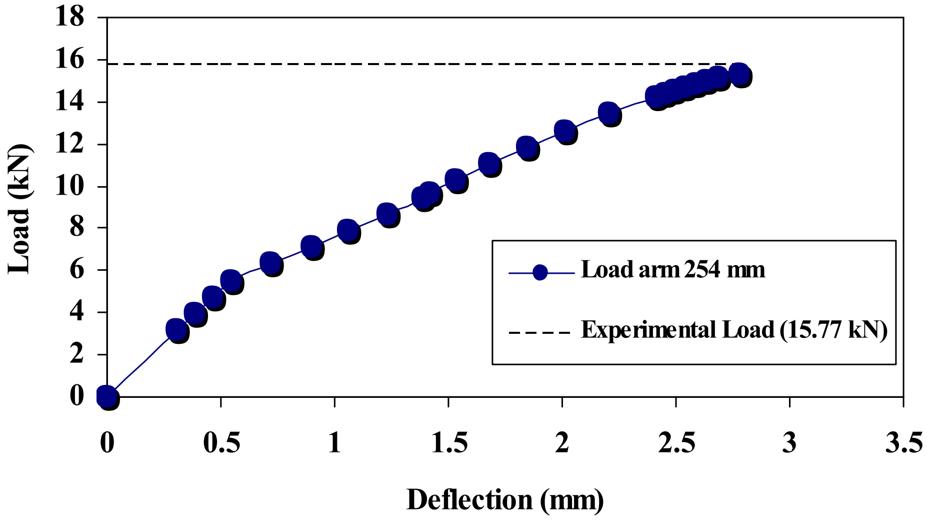
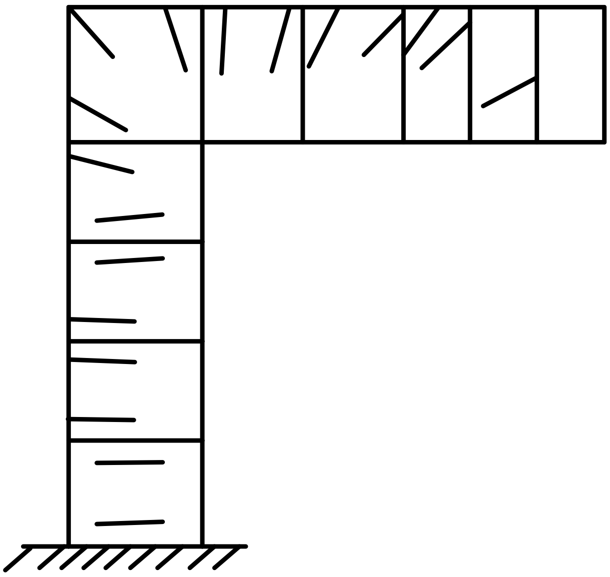


Fig.(6) Load-Deflection Curve for L-type Beam Column Connection .



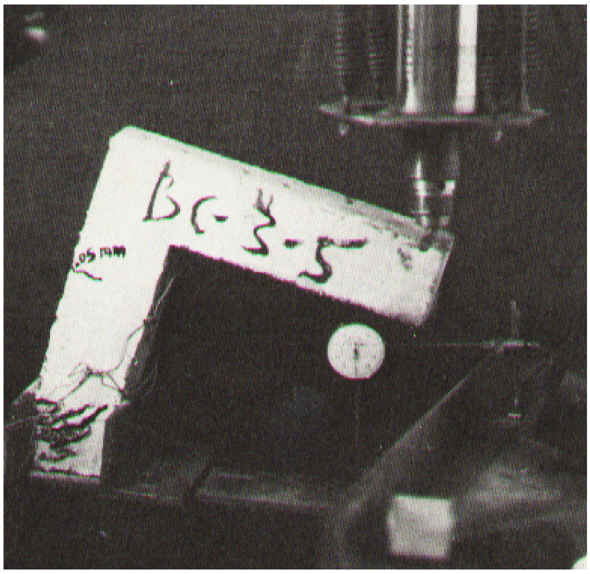


Fig.(7)- Typical Beam Column Connection after Test.

Fig.(8)- Predicted Crack Pattern of L-Type Beam Column Connection at Failure Load.

Knee-type connection

Sood and Gupta [17] tested a series of knee-type fibrous reinforced concrete connections. Two of these were chosen for the analysis. Fig.(9) shows the dimensions, details of reinforcement and the finite element mesh. The steel fibres were the same as those for the cross-type connection with a volume fractions of 0.6% and 0.8% with a corresponding cube strength of 23.5MPa and 24.1MPa respectively. Yield strengths of the reinforcement were 284MPa and 384MPa for #6 and #8 bars respectively. The numerical failure loads were respectively 23.3 kN and 24.99kN while the corresponding experimental failure loads were 23.54 kN and 25.01kN.

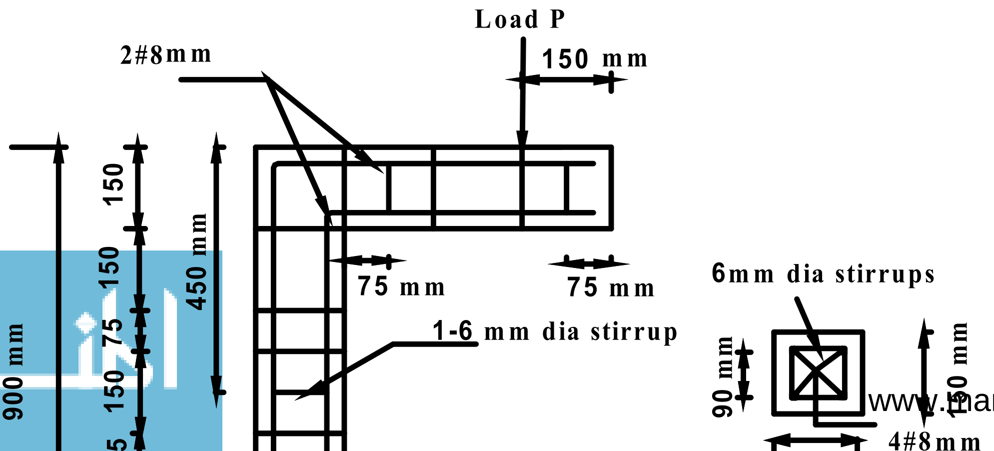
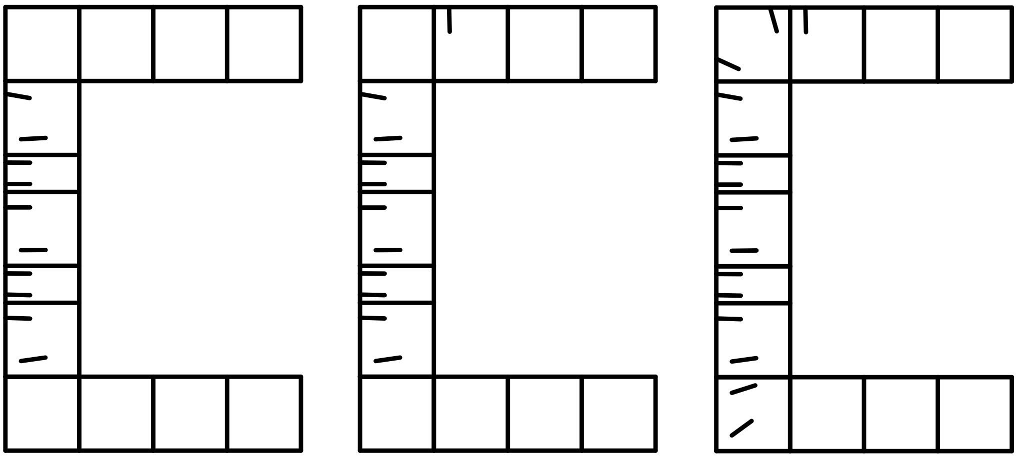
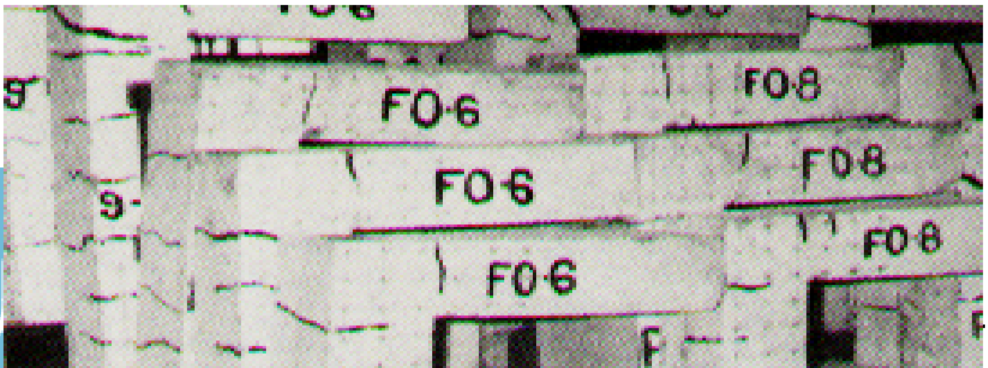


Fig. (10) shows the predicted crack pattern for the specimen with 0.6% fibres at three load levels up to failure and Fig. (11) shows the experimental cracks pattern which indicates a good agreement with the numerical one. Load –deflection curve of this specimen is shown in Fig. (12).



a)at Load(15.3)kN   b)at Load (19)kN   c)at Load(23.29)kN

Fig.(10) Predicted Crack pattern of Knee Type Beam Column Connection at Different Load Up to Failure.



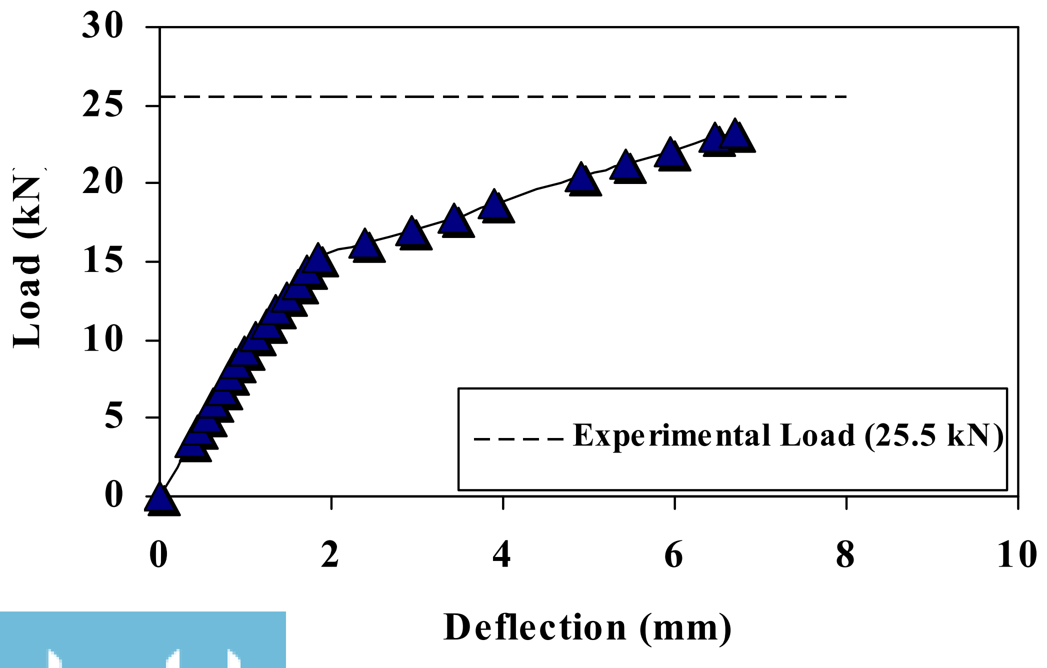


Fig.(12) Load Deflection Curve of Knee Type Beam Column Connection.

Table (2) Experimental and Numerical Failure Loads of the Analyzed Specimens

Example	Specimen No.	V_f %	Lever arm (mm)	Experimental load (kN)	Numerical load (kN)	Num./Exp. Load*
Cross Type	1	0.6	975	49.50	51.90	1.05
	2	1.0	975	75.02	73.50	0.98
L - Shape	1	0.6	254.0	15.77	14.50	0.92
	2	0.6	279.2	19.60	16.73	0.85
	3	0.6	304.4	15.83	15.36	0.97

Knee Type	1	0.6	450	23.54	23.30	0.99
	2	0.8	450	25.01	24.99	1.00

* Average = 0.966, C.O.V. = 6.6%

Conclusions

The adopted solution algorithm and the materials constitutive relationships for steel fibrous concrete gave a safe and realistic prediction of the behaviour and strength of the investigated fibrous reinforced concrete beam-column joints. The developed computer program can be used for a parametric study instead of conducting expensive experimental testing programmes. Refinement of the adopted relationships can result in a better prediction.

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